

WHAT IS CLAIMED IS:

1. An arithmetic performance attribution method for determining portfolio performance, relative to a benchmark, over multiple time periods t , where t varies from 1 to T , comprising the steps of:

- 5 (a) determining coefficients $(A + \alpha_t)$, where the values α_t are defined as

$$\alpha_t = \left[\frac{R - \bar{R} - A \sum_{k=1}^T (R_k - \bar{R}_k)}{\sum_{k=1}^T (R_k - \bar{R}_k)^2} \right] (R_t - \bar{R}_t),$$

where A has any predetermined value, R_t is a portfolio return for period t , \bar{R}_t is a benchmark return for period t , R is determined by

$$R = \left[\prod_{t=1}^T (1 + R_t) \right] - 1,$$

- 10 and \bar{R} is determined by

$$\bar{R} = \left[\prod_{t=1}^T (1 + \bar{R}_t) \right] - 1;$$

and

- (b) determining the portfolio performance as

$$R - \bar{R} = \sum_{t=1}^T (A + \alpha_t)(R_t - \bar{R}_t).$$

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2. The method of claim 1, wherein A is

$$A = \frac{1}{T} \left[\frac{(R - \bar{R})}{(1 + R)^{1/T} - (1 + \bar{R})^{1/T}} \right], \text{ where } R \neq \bar{R},$$

or for the special case $R = \bar{R}$:

$$A = (1 + R)^{(T-1)/T}.$$

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3. The method of claim 1, wherein $A = 1$.

4. The method of claim 1, wherein step (b) is performed by determining the portfolio performance as

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$$R - \bar{R} = \sum_{t=1}^T \sum_{i=1}^N (A + \alpha_t)(I_{it}^A + S_{it}^A),$$

where I_{it}^A is an issue selection for sector i and period t , and S_{it}^A is a sector selection for sector i and period t .

5 5. A computer system, comprising:

a processor programmed to perform an arithmetic performance attribution computation to determine portfolio performance, relative to a benchmark, over multiple time periods t , where t varies from 1 to T , by determining coefficients $(A + \alpha_t)$, where the values α_t are defined as

$$10 \quad \alpha_t = \left[\frac{R - \bar{R} - A \sum_{k=1}^T (R_k - \bar{R}_k)}{\sum_{k=1}^T (R_k - \bar{R}_k)^2} \right] (R_t - \bar{R}_t),$$

where A has any predetermined value, R_t is a portfolio return for period t , \bar{R}_t is a benchmark return for period t , R is determined by

$$R = \left[\prod_{t=1}^T (1 + R_t) \right] - 1,$$

and \bar{R} is determined by

$$15 \quad \bar{R} = \left[\prod_{t=1}^T (1 + \bar{R}_t) \right] - 1;$$

and determining the portfolio relative performance as

$$R - \bar{R} = \sum_{t=1}^T (A + \alpha_t)(R_t - \bar{R}_t); \text{ and}$$

a display device coupled to the processor for displaying a result of the arithmetic performance attribution computation.

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6. A computer readable medium which stores code for programming a processor to perform an arithmetic performance attribution computation to determine portfolio performance, relative to a benchmark, over multiple time periods t , where t varies from 1 to T , by determining coefficients $(A + \alpha_t)$, where the values α_t are defined as

$$25 \quad \alpha_t = \left[\frac{R - \bar{R} - A \sum_{k=1}^T (R_k - \bar{R}_k)}{\sum_{k=1}^T (R_k - \bar{R}_k)^2} \right] (R_t - \bar{R}_t),$$

where A has any predetermined value, R_t is a portfolio return for period t , \bar{R}_t is a benchmark return for period t , R is determined by

$$R = \left[\prod_{t=1}^T (1 + R_t) \right] - 1,$$

and \bar{R} is determined by

$$\bar{R} = \left[\prod_{t=1}^T (1 + \bar{R}_t) \right] - 1;$$

and determining the portfolio relative performance as $R - \bar{R} = \sum_{t=1}^T (A + \alpha_t)(R_t - \bar{R}_t)$.

7. A geometric performance attribution method for determining portfolio performance, relative to a benchmark, over multiple time periods t , where t varies from 1 to T , comprising the steps of:

determining attribution effects for issue selection $(1 + I_{it}^G)$ given by

$$1 + I_{it}^G = \frac{1 + w_{it} r_{it}}{1 + w_{it} \bar{r}_{it}} \Gamma_{it}^I,$$

and determining attribution effects for sector selection $(1 + S_{it}^G)$ given by

$$1 + S_{it}^G = \left(\frac{1 + w_{it} \bar{r}_{it}}{1 + \bar{w}_{it} \bar{r}_{it}} \right) \left(\frac{1 + \bar{w}_{it} \bar{R}_t}{1 + w_{it} \bar{R}_t} \right) \Gamma_{it}^S,$$

where r_{jt} is a portfolio return for sector j for period t , \bar{r}_{jt} is a benchmark return for sector j for period t , w_{jt} is a weight for r_{jt} , \bar{w}_{jt} is a weight for \bar{r}_{jt} , R is determined by

$$R = \left[\prod_{t=1}^T (1 + R_t) \right] - 1$$

and \bar{R} is determined by

$$\bar{R} = \left[\prod_{t=1}^T (1 + \bar{R}_t) \right] - 1;$$

and determining the portfolio performance as

$$\frac{1 + R}{1 + \bar{R}} = \prod_{t=1}^T \prod_{i=1}^N (1 + I_{it}^G)(1 + S_{it}^G).$$

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$$\Gamma_t^S = \left[\frac{1 + \tilde{R}_t}{1 + \bar{R}_t} \prod_{j=1}^N \left(\frac{1 + \bar{w}_{jt} \bar{r}_{jt}}{1 + w_{jt} \bar{r}_{jt}} \right) \left(\frac{1 + w_{jt} \bar{R}_t}{1 + \bar{w}_{jt} \bar{R}_t} \right) \right]^{1/N}.$$

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$$\Gamma_t^I = \Gamma_t^S = \Gamma_t = \left[\left(\frac{1+R_t}{1+\bar{R}_t} \right) \prod_{j=1}^N \frac{(1+\bar{w}_{jt}\bar{r}_{jt})(1+w_{jt}\bar{R}_t)}{(1+w_{jt}r_{jt})(1+\bar{w}_{jt}\bar{R}_t)} \right]^{\frac{1}{2N}}.$$

a processor programmed to perform a geometric performance attribution computation to determine portfolio performance, relative to a benchmark, over multiple time periods t , where t varies from 1 to T , by determining attribution effects for issue selection $(1 + I_{it}^G)$ given by

$$1 + I_{ii}^G = \frac{1 + w_{ii} r_{ii}}{1 + w_{ii} \bar{r}_{ii}} \Gamma_i^I, \quad$$

15 and determining attribution effects for sector selection $(1 + S_{ij}^G)$ given by

$$1 + S_{ii}^G = \left(\frac{1 + w_{ii} \bar{r}_{ii}}{1 + \bar{w}_{ii} \bar{r}_{ii}} \right) \left(\frac{1 + \bar{w}_{ii} \bar{R}_i}{1 + w_{ii} \bar{R}_i} \right) \Gamma_i^S, \quad (10)$$

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$$R = [\prod_{t=1}^T (1 + R_t)] - 1$$

and \bar{R} is determined by

$$\bar{R} = [\prod_{t=1}^T (1 + \bar{R}_t)] - 1,$$

and determining the portfolio performance as

$$\frac{1+R}{1+\bar{R}} = \prod_{t=1}^T \prod_{i=1}^N (1+I_{it}^G)(1+S_{it}^G);$$

and

- 5 a display device coupled to the processor for displaying a result of the geometric performance attribution computation.

11. The system of claim 10, wherein the values of Γ_t^I are

$$\Gamma_t^I = \left[\frac{1+R_t}{1+\bar{R}_t} \prod_{j=1}^N \left(\frac{1+w_{jt}\bar{r}_{jt}}{1+w_{jt}r_{jt}} \right) \right]^{1/N} \text{ and the values of } \Gamma_t^S \text{ are}$$

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$$\Gamma_t^S = \left[\frac{1+\bar{R}_t}{1+\bar{R}_t} \prod_{j=1}^N \left(\frac{1+\bar{w}_{jt}\bar{r}_{jt}}{1+\bar{w}_{jt}\bar{r}_{jt}} \right) \left(\frac{1+w_{jt}\bar{R}_t}{1+\bar{w}_{jt}\bar{R}_t} \right) \right]^{1/N}.$$

12. A computer readable medium which stores code for programming a processor to perform a geometric performance attribution computation to determine portfolio performance, relative to a benchmark, over multiple time periods t , where t varies from 1

- 15 to T , by determining attribution effects for issue selection $(1+I_{it}^G)$ given by

$$1+I_{it}^G = \frac{1+w_{it}r_{it}}{1+w_{it}\bar{r}_{it}} \Gamma_t^I,$$

and determining attribution effects for sector selection $(1+S_{it}^G)$ given by

$$1+S_{it}^G = \left(\frac{1+w_{it}\bar{r}_{it}}{1+\bar{w}_{it}\bar{r}_{it}} \right) \left(\frac{1+\bar{w}_{it}\bar{R}_t}{1+\bar{w}_{it}\bar{R}_t} \right) \Gamma_t^S,$$

where r_{jt} is a portfolio return for sector j for period t , \bar{r}_{jt} is a benchmark return for sector

- 20 j for period t , w_{jt} is a weight for r_{jt} , \bar{w}_{jt} is a weight for \bar{r}_{jt} , R is determined by

$$R = \left[\prod_{t=1}^T (1+R_t) \right] - 1$$

and \bar{R} is determined by

$$\bar{R} = \left[\prod_{t=1}^T (1+\bar{R}_t) \right] - 1; \text{ and determining the portfolio performance as}$$

$$\frac{1+R}{1+\bar{R}} = \prod_{t=1}^T \prod_{i=1}^N (1+I_{it}^G)(1+S_{it}^G).$$

13. The computer readable medium of claim 12, wherein the values of Γ_t^1 are

$$\Gamma_t^I = \left[\frac{1+R_t}{1+\bar{R}_t} \prod_{j=1}^N \left(\frac{1+w_{jt}\bar{r}_{jt}}{1+w_{jt}r_{jt}} \right) \right]^{1/N} \text{ and the values of } \Gamma_t^S \text{ are}$$

$$5 \quad \Gamma_t^S = \left[\frac{1+\bar{R}_t}{1+\bar{R}_t} \prod_{j=1}^N \left(\frac{1+\bar{w}_{jt}\bar{r}_{jt}}{1+\bar{w}_{jt}\bar{r}_{jt}} \right) \left(\frac{1+w_{jt}\bar{R}_t}{1+\bar{w}_{jt}\bar{R}_t} \right) \right]^{1/N}.$$

14. A geometric performance attribution method for determining portfolio performance, relative to a benchmark, over multiple time periods t , where t varies from 1 to T , comprising the steps of:

10 determining attribution effects $1+Q_{ijt}^G$ given by

$$1+Q_{ijt}^G = \prod_k \left(\frac{1+a_{ijt}^k}{1+b_{ijt}^k} \right) \Gamma_{ijt}^k,$$

where Γ_{ijt}^k are corrective terms that satisfy the constraint $\prod_{ij} (1+Q_{ijt}^G) = \frac{1+R_t}{1+\bar{R}_t}$, each of

a_{ijt}^k and b_{ijt}^k is a coefficient for attribution effect j , sector i , and period t , the coefficients a_{ijt}^k and b_{ijt}^k are obtained from arithmetic attribution effects $Q_{ijt}^A = \sum_k a_{ijt}^k - \sum_k b_{ijt}^k$ which

15 correspond to the attribution effects $1+Q_{ijt}^G$, R_t is a portfolio return for period t , \bar{R}_t is a benchmark return for period t , where R is determined by

$$R = \left[\prod_{t=1}^T (1+R_t) \right] - 1$$

and \bar{R} is determined by

$$\bar{R} = \left[\prod_{t=1}^T (1+\bar{R}_t) \right] - 1; \text{ and}$$

20 determining the portfolio performance as

$$\frac{1+R}{1+\bar{R}} = \prod_{t=1}^T \prod_{i=1}^N \prod_{j=1}^M (1+Q_{ijt}^G).$$

15. The method of claim 14, wherein $M = 2$, $1 + Q_{it}^G$ are attribution effects for issue election given by $1 + Q_{it}^G = \frac{1 + w_{it}r_{it}}{1 + w_{it}\bar{r}_{it}} \Gamma_t^I$, and $1 + Q_{it}^G$ are attribution effects for sector selection given by $1 + Q_{it}^G = \left(\frac{1 + w_{it}\bar{r}_{it}}{1 + \bar{w}_{it}\bar{r}_{it}} \right) \left(\frac{1 + \bar{w}_{it}\bar{R}_t}{1 + w_{it}\bar{R}_t} \right) \Gamma_t^S$,

5 where r_{it} is a portfolio return for sector i for period t , \bar{r}_{it} is a benchmark return for sector i for period t , w_{it} is a weight for r_{it} , \bar{w}_{it} is a weight for \bar{r}_{it} , the values of Γ_t^I are

$$\Gamma_t^I = \left[\frac{1 + R_t}{1 + \bar{R}_t} \prod_{i=1}^N \left(\frac{1 + w_{it}\bar{r}_{it}}{1 + w_{it}r_{it}} \right) \right]^{1/N}, \text{ and}$$

$$\text{the values of } \Gamma_t^S \text{ are } \Gamma_t^S = \left[\frac{1 + \bar{R}_t}{1 + \bar{R}_t} \prod_{i=1}^N \left(\frac{1 + \bar{w}_{it}\bar{r}_{it}}{1 + w_{it}\bar{r}_{it}} \right) \left(\frac{1 + w_{it}\bar{R}_t}{1 + \bar{w}_{it}\bar{R}_t} \right) \right]^{1/N}.$$

10 16. A computer system, comprising:

a processor programmed to perform a geometric performance attribution computation to determine portfolio performance, relative to a benchmark, over multiple time periods t , where t varies from 1 to T , by determining attribution effects $1 + Q_{ijt}^G$ given by

$$15 \quad 1 + Q_{ijt}^G = \prod_k \left(\frac{1 + a_{ijt}^k}{1 + b_{ijt}^k} \right) \Gamma_{ijt}^k,$$

where Γ_{ijt}^k are corrective terms that satisfy the constraint $\prod_{ij} (1 + Q_{ijt}^G) = \frac{1 + R_t}{1 + \bar{R}_t}$, each of

a_{ijt}^k and b_{ijt}^k is a coefficient for attribution effect j , sector i , and period t , the coefficients a_{ijt}^k and b_{ijt}^k are obtained from arithmetic attribution effects $Q_{ijt}^A = \sum_k a_{ijt}^k - \sum_k b_{ijt}^k$ which correspond to the attribution effects $1 + Q_{ijt}^G$, R_t is a portfolio return for period t , \bar{R}_t is a

20 benchmark return for period t , R is determined by

$$R = \left[\prod_{t=1}^T (1 + R_t) \right] - 1$$

and \bar{R} is determined by

$$\bar{R} = \left[\prod_{t=1}^T (1 + \bar{R}_t) \right] - 1, \text{ and}$$

determining the portfolio performance as $\frac{1+R}{1+\bar{R}} = \prod_{t=1}^T \prod_{i=1}^N \prod_{j=1}^M (1 + Q_{ijt}^G)$; and

5 a display device coupled to the processor for displaying a result of the geometric performance attribution computation.

17. The system of claim 16, wherein $M = 2$, $1 + Q_{it}^G$ are attribution effects for issue election given by $1 + Q_{it}^G = \frac{1 + w_{it} r_{it}}{1 + w_{it} \bar{r}_{it}} \Gamma_t^I$, and $1 + Q_{i2t}^G$ are attribution effects for sector selection given by $1 + Q_{i2t}^G = \left(\frac{1 + w_{it} \bar{r}_{it}}{1 + \bar{w}_{it} \bar{r}_{it}} \right) \left(\frac{1 + \bar{w}_{it} \bar{R}_t}{1 + w_{it} \bar{R}_t} \right) \Gamma_t^S$,
 10 where r_{it} is a portfolio return for sector i for period t , \bar{r}_{it} is a benchmark return for sector i for period t , w_{it} is a weight for r_{it} , \bar{w}_{it} is a weight for \bar{r}_{it} , the values of Γ_t^I are $\Gamma_t^I = \left[\frac{1 + R_t}{1 + \bar{R}_t} \prod_{i=1}^N \left(\frac{1 + w_{it} \bar{r}_{it}}{1 + w_{it} r_{it}} \right) \right]^{1/N}$, and
 the values of Γ_t^S are $\Gamma_t^S = \left[\frac{1 + \bar{R}_t}{1 + \bar{R}_t} \prod_{i=1}^N \left(\frac{1 + \bar{w}_{it} \bar{r}_{it}}{1 + w_{it} \bar{r}_{it}} \right) \left(\frac{1 + w_{it} \bar{R}_t}{1 + \bar{w}_{it} \bar{R}_t} \right) \right]^{1/N}$.

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18. A computer readable medium which stores code for programming a processor to perform a geometric performance attribution computation to determine portfolio performance, relative to a benchmark, over multiple time periods t , where t varies from 1 to T , by determining attribution effects $1 + Q_{ijt}^G$ given by

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$$1 + Q_{ijt}^G = \prod_k \left(\frac{1 + a_{ijt}^k}{1 + b_{ijt}^k} \right) \Gamma_{ijt}^k,$$

where Γ_{ijt}^k are corrective terms that satisfy the constraint $\prod_{ij} (1 + Q_{ijt}^G) = \frac{1 + R_t}{1 + \bar{R}_t}$, each of a_{ijt}^k and b_{ijt}^k is a coefficient for attribution effect j , sector i , and period t , R_t is a portfolio return for period t , the coefficients a_{ijt}^k and b_{ijt}^k are obtained from arithmetic attribution effects $Q_{ijt}^A = \sum_k a_{ijt}^k - \sum_k b_{ijt}^k$ which correspond to the attribution effects $1 + Q_{ijt}^G$, \bar{R}_t is a

- 5 benchmark return for period t , R is determined by $R = [\prod_{t=1}^T (1 + R_t)] - 1$, and \bar{R} is determined by $\bar{R} = [\prod_{t=1}^T (1 + \bar{R}_t)] - 1$, and determining the portfolio performance as

$$\frac{1 + R}{1 + \bar{R}} = \prod_{t=1}^T \prod_{i=1}^N \prod_{j=1}^M (1 + Q_{ijt}^G).$$

19. The computer readable medium of claim 18, wherein $M = 2$, $1 + Q_{it}^G$ are

- 10 attribution effects for issue election given by $1 + Q_{it}^G = \frac{1 + w_{it} r_{it}}{1 + w_{it} \bar{r}_{it}} \Gamma_t^I$, and $1 + Q_{i2t}^G$ are

attribution effects for sector selection given by $1 + Q_{i2t}^G = \left(\frac{1 + w_{it} \bar{r}_{it}}{1 + \bar{w}_{it} \bar{r}_{it}} \right) \left(\frac{1 + \bar{w}_{it} \bar{R}_t}{1 + w_{it} \bar{R}_t} \right) \Gamma_t^S$,

where r_{it} is a portfolio return for sector i for period t , \bar{r}_{it} is a benchmark return for sector i for period t , w_{it} is a weight for r_{it} , \bar{w}_{it} is a weight for \bar{r}_{it} , the values of Γ_t^I are

$$\Gamma_t^I = \left[\frac{1 + R_t}{1 + \bar{R}_t} \prod_{i=1}^N \left(\frac{1 + w_{it} \bar{r}_{it}}{1 + w_{it} r_{it}} \right) \right]^{1/N}, \text{ and}$$

- 15 the values of Γ_t^S are $\Gamma_t^S = \left[\frac{1 + \bar{R}_t}{1 + \bar{R}_t} \prod_{i=1}^N \left(\frac{1 + \bar{w}_{it} \bar{r}_{it}}{1 + w_{it} \bar{r}_{it}} \right) \left(\frac{1 + w_{it} \bar{R}_t}{1 + \bar{w}_{it} \bar{R}_t} \right) \right]^{1/N}$.